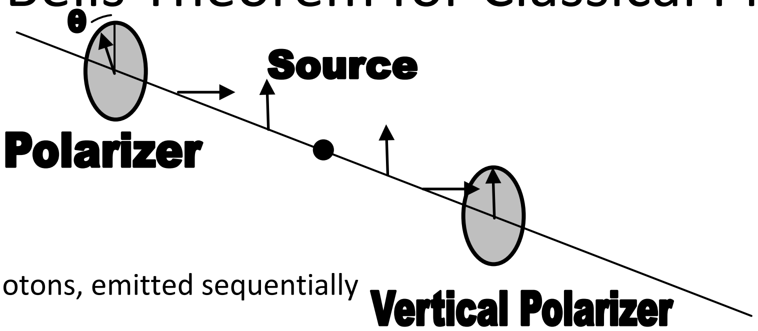
Bells Theorem for Classical Physics



System

Source is unpolarized photons, emitted sequentially

Polarizer on the right is oriented vertically. Polarizer on the left is at angle θ from vertical.

Solution

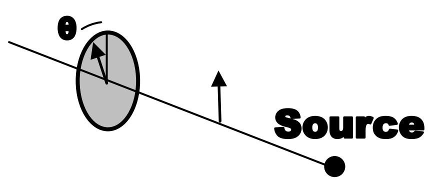
The Quantum Bell setup is the following. A pair of photons is emitted from a single source and propagates left and right. The photon pair shares initial polarization angle. A detector sits behind the polarizers and only counts when a pair of photons makes it through on both sides. The probability the detector will detect both photons will be expressed as the following.

$$P\big((A \cap B)|C\big) = P\big(A|(B \cap C)\big)P(B|C)$$

A represents the photon getting through the polarizer on the left. B represents the photon passing on the right. C represents the source emitting the same photon left and right. The second factor is quite simple. It represents the probability a photon passes through a vertical polarizer on the right. That is half, since the source is unpolarized and the incident photon will be horizontal or vertical with equal probability.

$$P(B|C) = \frac{1}{2}$$

The first factor in words is the probability that the photon passes through the polarizer on the left given that the photon shares polarization with the photon on the right and the photon on the right passed. This condition can only be true, if the incident photon on the left is also vertically polarized. The first factor is illustrated in the following illustration.



The condition B and C necessitate a vertically polarized photon incident on the left polarizer, which is at angle θ from vertical. Don't believe determining conditional probability outlines a real physical mechanism. Of course, horizontally polarized photons on the left still exist. The probability the photon passes is a straight forward application of Malus's law, so the first factor becomes the following, and the joint probability can be found subsequently.

$$P\big(A|(B \cap C)\big) = cos^2(\theta) \rightarrow P\big((A \cap B)|C\big) = \frac{1}{2}cos^2(\theta)$$

Most authors neglect the fact that the source emits the same photon left and right, so I had to make that obvious. When acknowledged, it shows how the system is dependent on C both causally and probabilistically. This conditional dependence on C for A and B also leaves A and B probabilistically dependent, but not causally linked. To be abundantly clear, I never assumed a photon travels left then right or vice versa.