Modeling Insertion Loss Through Properties of Fractals

Background

Modeling insertion loss is important, to be able to predict channel performance. The losses are dielectric and conductor. Dielectric loss is well understood and easily modeled, if provided with accurate loss tangent, dielectric constant, and cross sectional geometry. Conductor loss is less intuitive to understand and has been a struggle to predict. This paper suggests a better way to model conductor loss. It calls attention to problems with the current state of the art Huray model. Examples are provided.

The Fractal Model

This model assumes self similarity of the surface under magnification. This is common for surfaces created from natural processes such as chemical etching. The model suggests the effective length of the conductor grows as a function of frequency. As the skin depth decreases the current has a longer length to travel, since it's bound closer to the surface. An illustration of this follows.

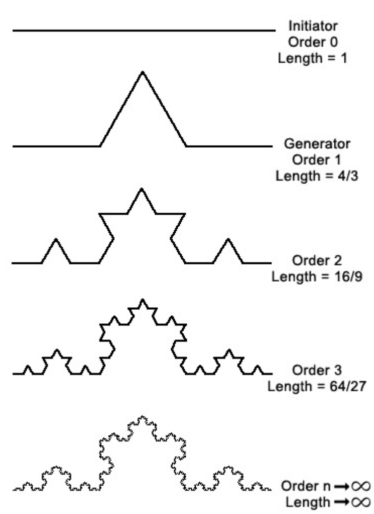


Figure 1. The effective length the current travels as skin depth decreases

The reader is expected to imagine the current being forced into an increasingly thin sheath along an infinitely complex fractal surface. For larger skin depths the current just goes round the smaller features of the fractal. A property of fractal surfaces is that when the standard length used to measure the surface decrease the length of the surface increases. The trend, when graphed on log axes is linear. The slope of the line is called the Fractal Dimension. The effective length with respect to granular measurement length can be expressed as follows

$$log(N) = Dlog(1/\varepsilon)$$

Here N is the effective length. D is the fractal dimension, and epsilon is the standard length used to measure the surface. Here the skin depth can be substituted for the standard length. The effective length will be N. Taking this into account, a model for the conductor loss coefficient with respect to skin depth can be created. It follows.

$$log(N) = Dlog\left(\frac{1}{\varepsilon}\right), N + 1 = K, \varepsilon \approx \delta$$

$$log(N) = log\left(\frac{1}{\delta^D}\right), N + 1 = K \rightarrow N = \frac{1}{a\delta^D} \rightarrow \underline{K = \frac{1}{\delta^D} + 1}$$

The model is very simple. The roughness can be modeled by a monomial. In the real world there will need to be scale factors, but this is a good starting point. Allowing for fitting the model has only 3 parameter. The expression follows. B is a constant very close to 1. a is a scaling factor, and D is the Fractal dimension of the conductor surface. Sigma is the skin depth which is a function of frequency.

$$K = \frac{a}{\delta^D} + b$$

This fits into the common loss model in a very common way. It just replaces the Huray model to model surface roughness. The common loss equation follows.

$$\alpha_{total} = \alpha_{dielectric} + K\alpha_{Bulk\ Conductor}$$

As a practical example, a few coupons had their Ks derived and fitted to the fractal model. A graph of this follows.

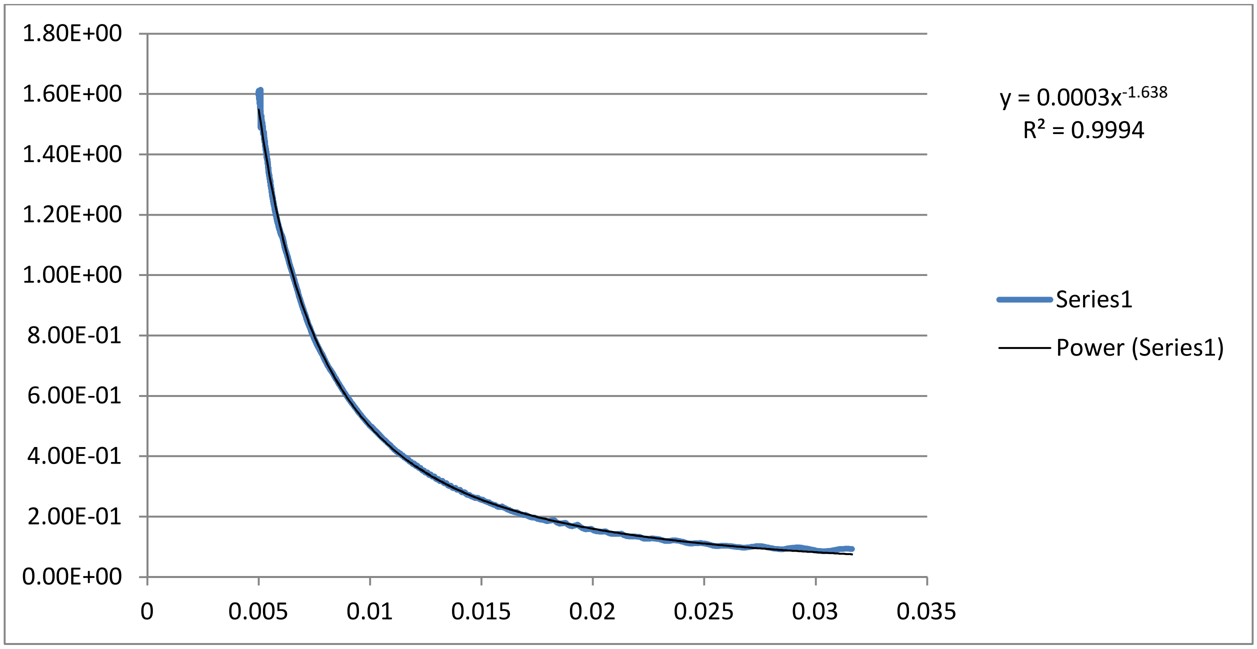


Figure 2. K is plotted as a function of skin depth

Here is a VNA measurement that went up to 40GHz. K is plotted with respect to the skin depth for copper at that frequency. From the regression it is seen that this model is a great fit. The coefficient of determination has 3 9s which is unheard of for lab measurements.

This model is causal. A proof is not forthcoming, but the function of conductor loss was created in Octave and its Hilbert transform was calculated. The Hilbert transform was the imaginary part. A picture of this follows.

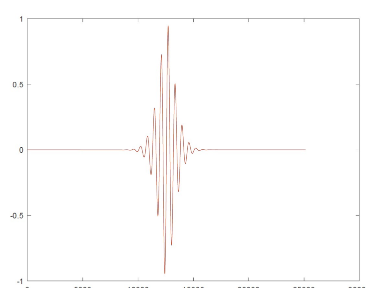


Figure 3. Red is the model and blue is the Hilbert transform

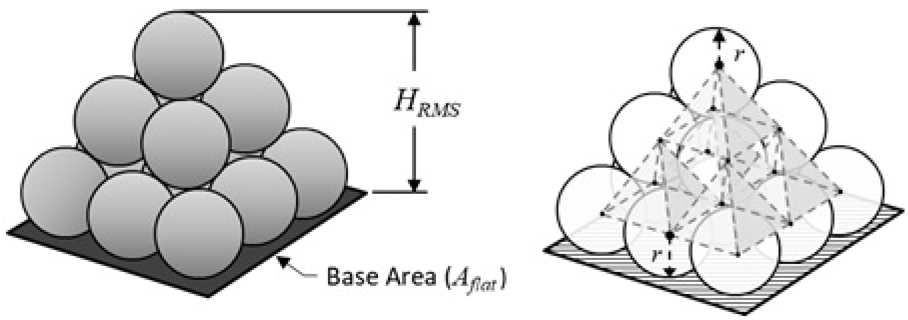
From the calculation it can be seen that the fractal model of conductor loss is causal to a very large degree. There is no proof, so this isn't proven, but it's sufficiently demonstrated. Here the fractal dimension was 3/2.

The Huray Model

I'll simplify the Huray model for this comparison. The Huray model is the following. For every term the Huray model is fitted with the number of spheres and the radius of those spheres

$$K = b + \sum_{i=0}^{\infty} a_i e^{-\frac{\delta}{r_i}}$$

It should be noted initially this model can model anything given enough coefficients and radii. The reality is that going through the trouble to find more than one term in the expansion is rarely done. This reduces to the cannon or snowball model. It follows.



$$K = b + ae^{-\frac{\delta}{r}}$$

Here b is a constant close to 1. a is a scale factor proportional to the number of spheres and r is a radius of those balls. The problem with this model is that it saturates shortly after sigma approaches the radius. This has not been observed in an insertion loss plot up to any frequency. A fitting with a real measurement of K follows.

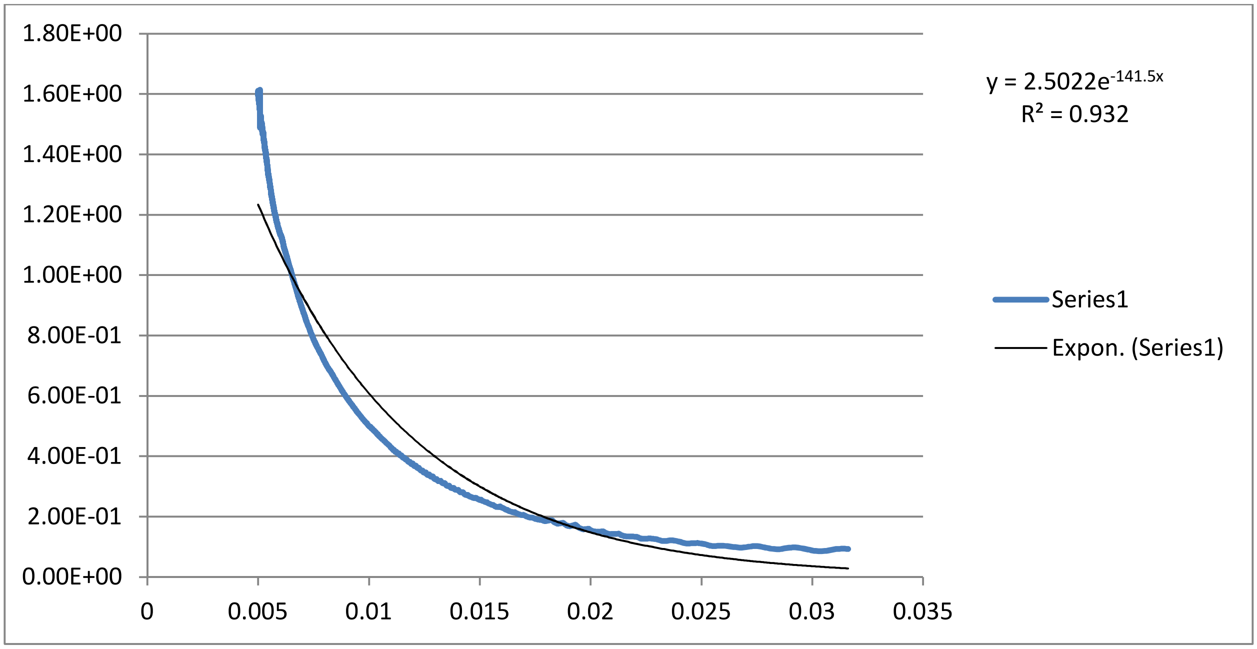


Figure 4. An exponential model of K

As seen here, the exponential Huray model fits much more poorly than the fractal model. The coefficient of determination is significantly lower.

The Huray model is not causal. Using it can lead to aphysical results. There is a special form of the Huray model that has been derived to overcome this problem in recent years. This causal model is somewhat more unweildly.

Comparison of Huray and Fractal Model

Fitting Physical Reality

For the same number of fitting constants the Fractal model is orders of magnitude more accurate.

Number of Fitting Parameters

Clearly, if there is an infinity of fitting parameters the Huray model will be perfect, but what is the point of using a model, then? Each model has 3 parameters in its basic commonly used form

Causality

Causality is important when creating electrical models in simulation environments, so that the results don't become aphysical. The fractal model has causality built into it. The Huray model can be made to be causal, but only after more complexity is added.

Speed of Calculation

There doesn't seem to be much of a difference between the Huray and fractal model in terms of computational complexity, but in the end the fractal model is just a monomial, so it seems marginally less complex and easier to calculate.

Conclusion

The fractal model is better in every sense and is more intuitive. To reiterate, the fractal model assumes the current will be confined to a smaller and smaller section along the surface. While this happens, the current has a longer and longer path to travel down the channel. The Huray model assumed there are a number of spheres that lay around and will increase the surface area. The Huray model has a theoretical bottleneck very quickly. It's clear the number of balls chosen per area and their radii will breakdown quickly at higher frequency. There will always be more balls and their numbers will always increase under magnification. For the Huray model to continue its relevance, the user is expected to keep creating new terms, which became unpopular after one term. The fractal model assumes an infinity of balls and radii from the beginning. The fractal model only makes one assumption that is the surface has a fractal nature, which in most circumstances seem to be very accurate. In a wide band sense this will have a lot of longevity.