Polarizer Theory with Applications to Bell Type Experiments

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Contributions

A.R. contributed the classical derivation; C.R. contributed atomic theory; T.N. contributed knowledge of electromagnetism

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Abstract

A comprehensive classical theory of polarizers is developed. This theory is in accordance with Malus's law, Maxwell's equations, and Young's double-slit experiment. The theory of light and polarizers is extended to the Bell type experimental results. A means of simulating the experiments is provided. Simulating Bell type experiments, the solution and theory violate the CHSH inequality.

Introduction

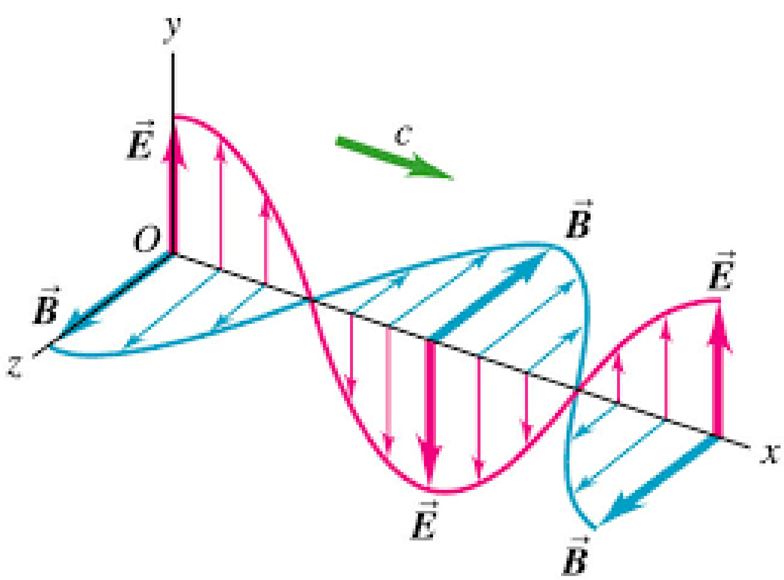
It's come to our attention that few understand the classical theory of light and polarizers, which is unfortunate because it provides a causal, conservative, and repeatable means of understanding the results of Bell type experiments without entanglement. No hidden variables are required to understand the Bell type experiments, in fact, the initial polarization angle is all that is necessary. Malus discovered a law that governs polarizer transmission. Young's experiment demonstrated light is a wave. Later Maxwell demonstrated that light is a propagating form of electromagnetism.

Theory

Light

According to Maxwell, light is a propagating wave of transverse electric and magnetic fields.

Fig. 1: Light as EM radiation



In the above illustration the electric field is aligned with the y axis. The magnetic field is aligned with the z axis. The fields propagate at the speed of light in the x direction. Maxwell's derivation that proves light is is easy to follow and intuitive and can be found elsewhere. From Faradays and Amperes law it can be shown that EM fields can be described with a wave equation. This result agrees with Young's double-slit experiment where light is demonstrated to interfere with itself constructively and destructively.

Light is not a particle, although it's convenient to think of the energy and momentum light carries as being discretely distributed when it is absorbed or emitted by an atom. This discretization is a consequence of the discrete nature of the atom, being composed of a certain number of components. In free space a quantum of light makes no sense. A quick disproof of the quantum of light is the fact that a discrete distribution can not be described by a finite band or discrete frequency. With one frequency one can only describe an infinite sinusoid. This is basic fourier analysis.

Polarized Light

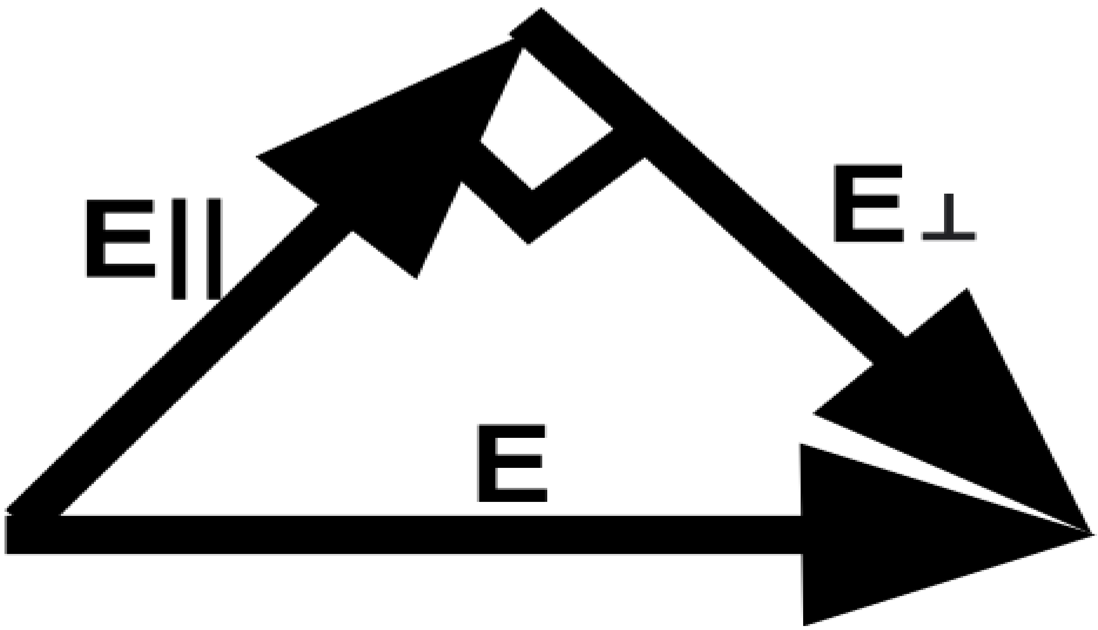
The illustration above is one of polarized light. There is only one component of the electric field in the y direction, so that is an example of linearly polarized light. If the solution in the illustration were added with a similar solution but with the electrical component on the z axis, then that would be unpolarized light. Unpolarized light is light that has equal components of electric or magnetic field in both transverse directions. Being a wave, light can be decomposed or composed in any way. Light should always be thought of as a superposition. Even in the illustration there exists infinite ways to decompose that simple example. A decomposition follows, assuming j-hat for the electric field.

$$E = \hat{j} = \hat{j} + \hat{k} - \hat{k} \tag{1}$$

This seems facetious or trivial but is very critical for understanding. An electric field along the y (or j-hat) direction can be thought of as a composition of one component of electric field in the y and z (k-hat) direction and one in the negative z direction. The vector components still add up to a unit of electric field in the y direction, so both representations are equivalent. The key to understanding classical light is that it is a superposition. However some superpositions are more useful than others.

A helpful way to look at polarized light, is to think of it as being composed of perpendicular components. An illustration follows.

Fig. 2: Perpendicular Components of Polarized Light



Given a polarization of light E, represented by a vector, there can always be found two perpendicular components E|| and E⊥ that add up to E. The distinction between the parallel and perpendicular component will become relevant later. The components will be derived now, with the polarization going back to being a unit along the y-axis, for expedience. Assume you pick any point in the y-z plane that represents the line E||.

$$E|| = yt\hat{j} + zt\hat{k} \tag{2}$$

A is a collection of all point that make up the line A with parameter t. To find the perpendicular component E⊥ that will sum with E|| to get E, the distance from any point on E|| to E needs to be minimized. This is a consequence of the fact that the minimum distance between a line and a point is always along a line perpendicular to E||.

$$|E\perp|^2 = (yt - 1)^2 + (zt)^2 \tag{3}$$

$$\frac{|E\perp|^2}{dt} = 2(yt - 1)y + 2(zt)z = 0 \tag{4}$$

$$t = \frac{y}{(y^2+z^2)} \tag{5}$$

Substituting the minimized t back into E|| solves for E|| in terms of the points y and z

$$E|| = \frac{y^2\hat{j}}{(y^2+z^2)} + \frac{yz\hat{k}}{(y^2+z^2)} \tag{6}$$

$$|E|||^2 = \frac{y^4}{(y^2+z^2)^2} + \frac{y^2z^2}{(y^2+z^2)^2} \tag{7}$$

$$|E|||^2 = \frac{y^4+y^2z^2}{(y^2+z^2)^2} \tag{8}$$

$$|E|||^2 = \frac{y^2(y^2+z^2)}{(y^2+z^2)^2} \tag{9}$$

$$|E|||^2 = \frac{y^2}{(y^2+z^2)} \tag{10}$$

Then convert to polar coordinates

$$y = rcos\theta, z = rsin\theta \tag{11}$$

$$|E|||^2 = \frac{r^2cos^2\theta}{r^2} \tag{12}$$

$$E|| = cos\theta \tag{13}$$

This is the common classical result for finding the magnitude of a force along a given direction, given theta is the angle between the incidence and the only allowable direction of motion, which makes intuitive sense. When solving problems like the force of gravity on a ball on a wedge, what is actually being done is solving for the ratio of the incident force in the direction along the direction of motion to the original. This represents the component of the incident that isn't off in a perpendicular direction. This point will come up later, when investigating polarizers.

Another point is that when solving for the equation of the electrical component E, with perpendicular E|| and E⊥

$$E = E|| + E\perp \tag{14}$$

Given that A is perpendicular to B the vector magnitudes will form a right triangle, and the following pythagorean relationship can be made.

$$|E|^2 = |E|||^2 + |E\perp|^2 \tag{15}$$

The vector quantities were electric field amplitudes. The field amplitude squared is proportional to the intensity, so the following can be written.

$$I_E = I_{E||} + I_{E\perp} \tag{16}$$

This shows that the solution conserves energy.

To finish this section, plotting the cross section of polarized radiation by radian is instructive.

Fig. 3: Cross Section of Polarized Light

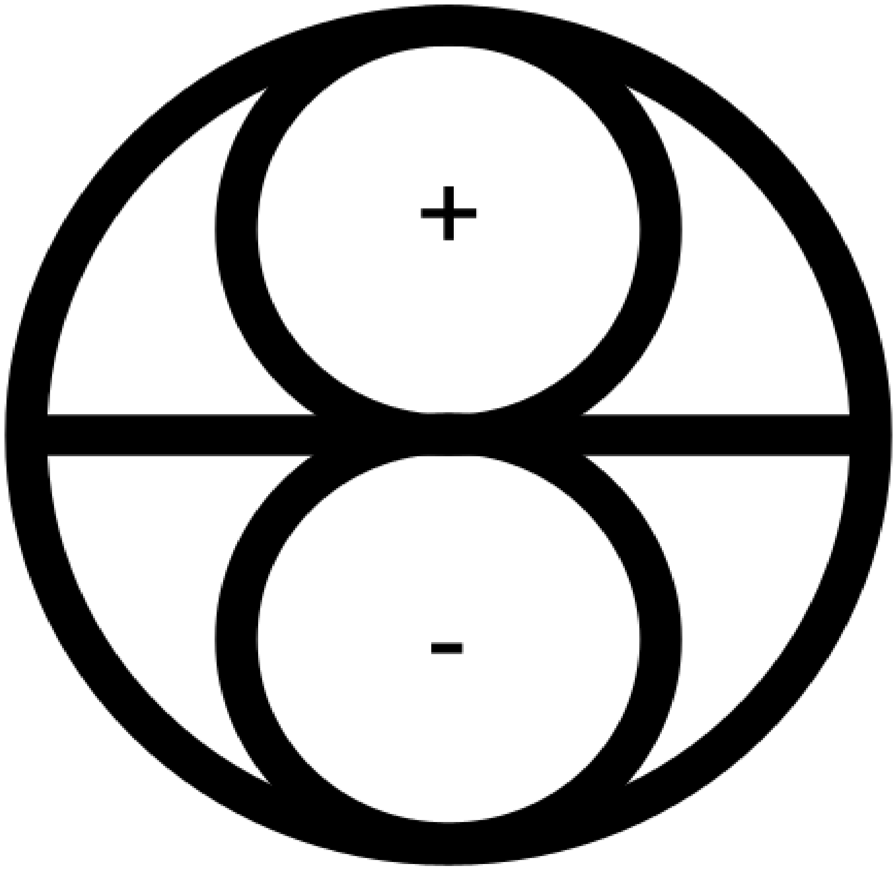


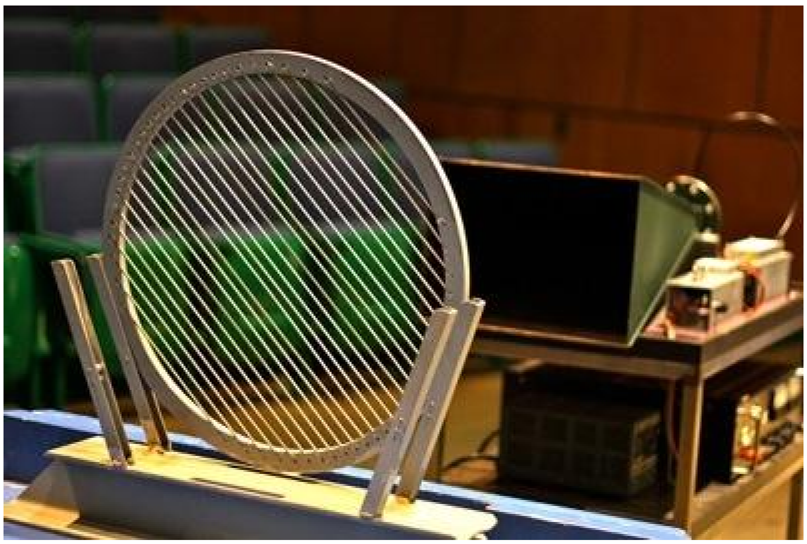
Figure 3 illustrates the cross section of polarized light.

This is a plot of the magnitude of A in polar coordinates. When the vector E|| is nearly parallel to E there is a very small component perpendicular to E|| and E|| is nearly E. When E|| is perpendicular to E then the magnitude of E|| goes to zero. The negative and positive lobe just denote where you'd find the head or tail of E||(spin). The sign will become useful later when calculating correlations for Bell type experiments where measurements on spin will be negative or positive.

Polarizers

Polarizers block the component of light perpendicular to their axis. Polarizers are anisotropic materials. There is a significant difference in how they react to electric or magnetic fields oriented in different directions. There can be macroscopic polarizers that work on microwaves, and there can be microscopic polarizers that work on visible light. Below is a representation of a macroscopic polarizer which is useful for illustration.

Fig. 4: Macroscopic Polarizer for Microwaves



This gives a good sense of how a polarizer works classically. Electric fields react to the presence of conductors. They cause currents to form. When an electric field is parallel is it clear it will propagate differently than when it is perpendicular to the wires.

When solving for the ratio of radiation that transmits through a polarizer, it's helpful to think of what polarized light is from the previous section.   If the component of the incident light along $E\perp$ is blocked then only $E\|$ remains. Polarizers transmit polarized light.

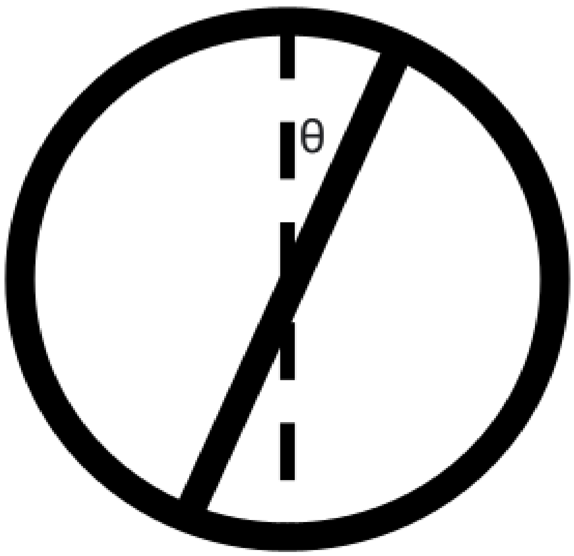
Metrology

Electromagnetic waves are measured by antennas. These antennas, like polarizers, can be microscopic or macroscopic. A picture of a macroscopic antenna follows.

Fig. 5: Macroscopic Antenna for Microwaves

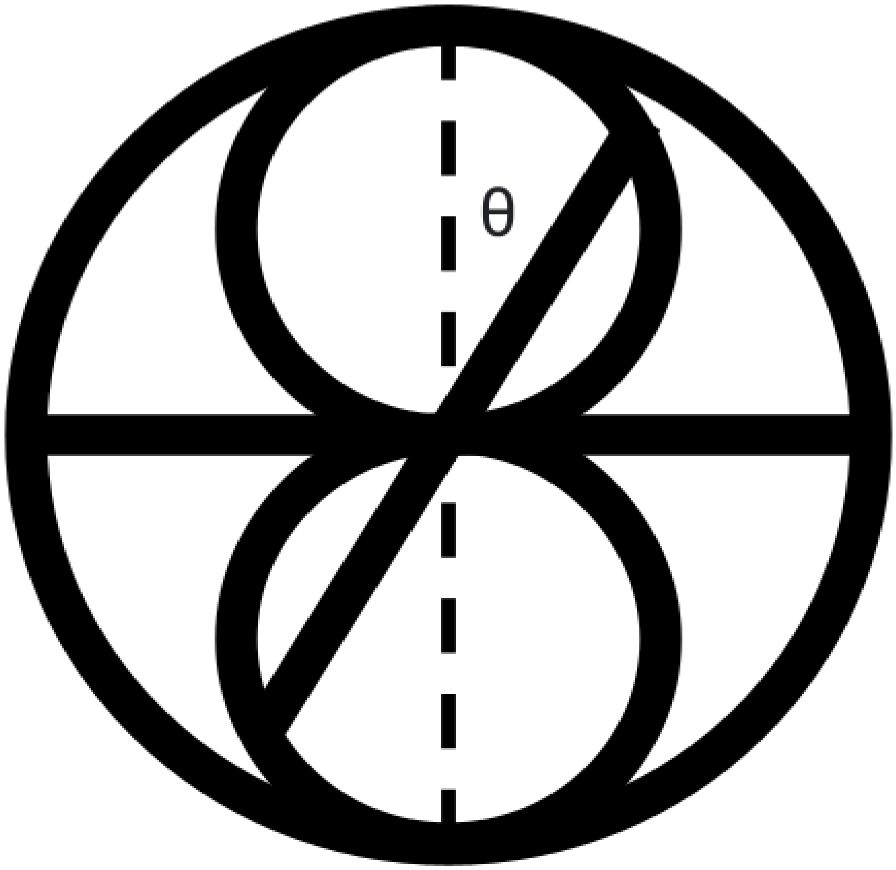
Antennas will absorb the component of the electromagnetic field parallel to them. An antenna will not absorb a component perpendicular to itself. From this a few basic observations can be made. An antenna will measure a constant EM field if it is constrained to swivel around in the transverse plane of unpolarized EM radiation. Pictographically unpolarized light can be represented by a circle, since it has an equivalent antenna reading at all angles.

Fig. 6: Antenna in unpolarized light



In every direction the antenna, represented by the bar, rotates the antenna will have the same reading. This indicates the measurement of unpolarized light. The situation becomes a little less trivial when measuring polarized light. When measuring polarized light, the component of the EM radiation parallel to the antenna is not constant, nor will it have a discrete distribution. Polarized light has an electric component as derived in the Polarized Light section. An illustration of an antenna measuring polarized light follows.

Fig. 7: Antenna Measuring Polarized Light



It was found previously that a full unit of EM amplitude was not found at all angles for polarized light. When the angle theta is swept the antenna will only be able to absorb a component proportional to the cosine of theta, since that is the component parallel to the antenna.

An interesting question is “what is the ratio of the unpolarized to polarized light?” This would answer the question of how much of the intensity is blocked by the presence of a polarizer. The answer, how the illustrations are presented now, would lead to an answer that is dependent on theta, or the position of the antenna with respect to the polarization, but first the total power should be found. Total power can be represented by summing up all the intensity at all angles. That can be represented as follows.

$$I_{total} = \int_{0}^{2\pi} I_{\theta} d\theta \quad (17)$$

The intensity is the amplitude of the electric field squared, which leads to the following, when expressing the parallel component in polar coordinates.

$$I_{total} = \int_{0}^{2\pi} r(\theta)^2 d\theta \quad (18)$$

This result is interesting because it's just an area integral in polar coordinates. In fact walking the double integral back one step leads to the following.

$$I_{total} = 2\int_{0}^{2\pi}\int_{0}^{r} r dr d\theta = 2Area \quad (19)$$

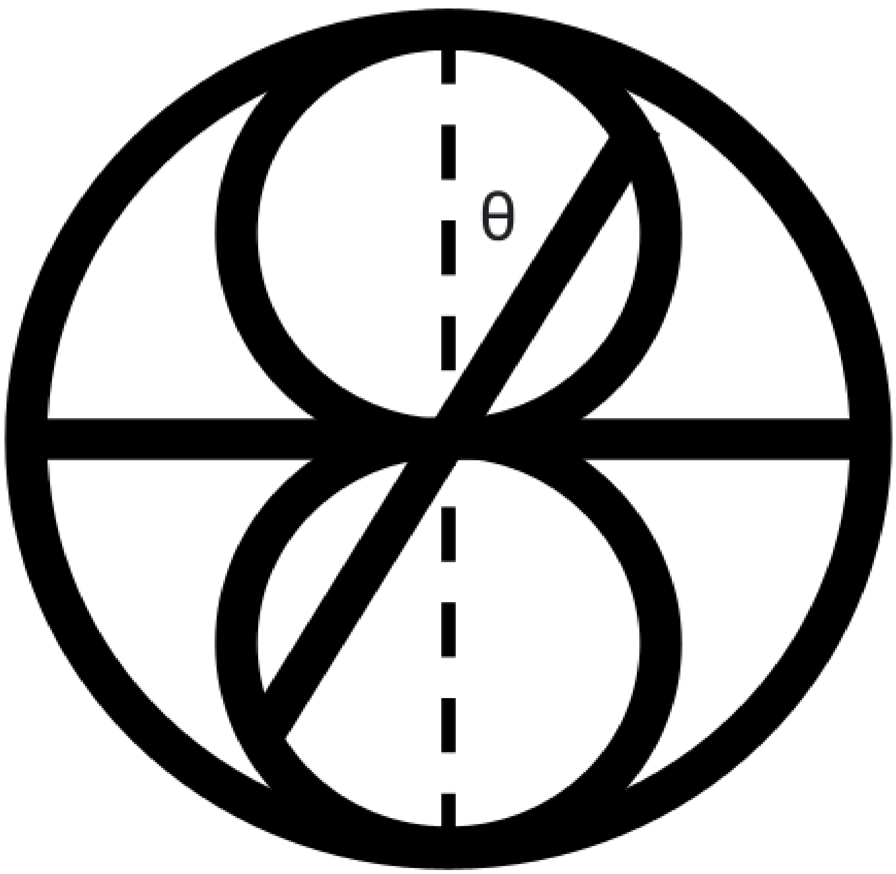
If the unpolarized lights area was divided by the polarized lights area, it would represent the ratio of unpolarized to polarized light intensity. This can be expressed as follows, if r is assumed to be the max radius and the two smaller circles have a radius half r.

$$\frac{I_{polarized}}{I_{unpolarized}} = \frac{A_{Polarized}}{A_{unpolarized}} = \frac{2\pi\left(\frac{r}{2}\right)^2}{\pi r^2} = \frac{1}{2} \quad (20)$$

This is a very critical result. It means that if a polarizer is placed between a ray of unpolarized light it will diminish the total intensity by half. This is the result of Malus.

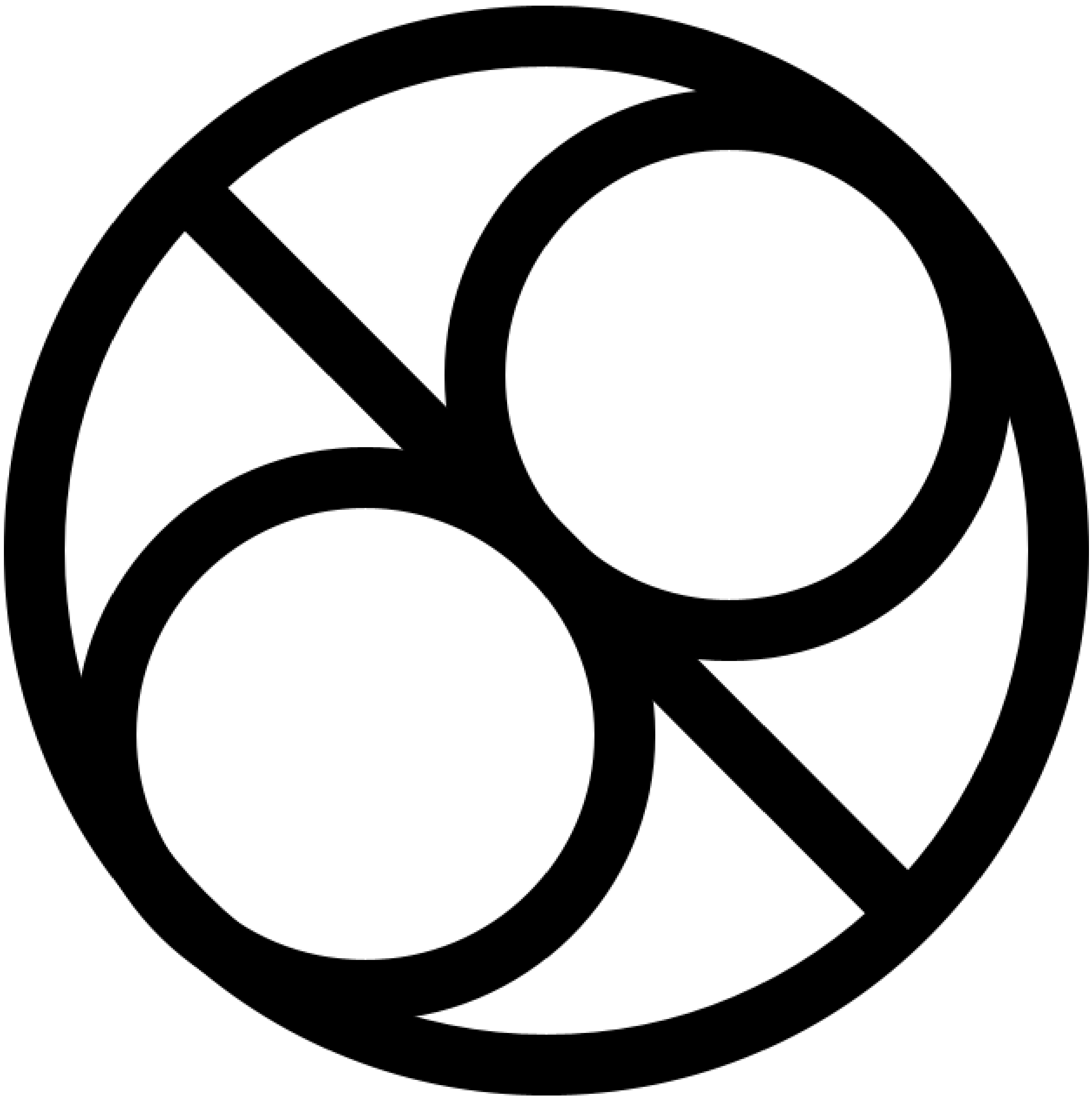
Another important question is “if a polarizer is placed after that polarizer, then what would be the result?” To answer this question the function of a polarizer is considered. A polarizer blocks all electric field components in the direction perpendicular to its axis. The solution is the same as the one for an antenna. A diagram follows, where the bar is now a polarizer.

Fig. 8: Cascaded Polarizer at Angle Theta



Inspecting the diagram it becomes clear. The curve is the solution for the component of the original parallel to its axis and none perpendicular. That means the curve is the solution for a cascaded polarizer, because it's the only solution with the perpendicular component blocked. What remains is a component of the original electric field parallel to the axis of the polarizer. The same process from the first section can be repeated to get the solution for transmitted lights pattern from the second polarizer.

Fig. 9: Transmitted Light from Second Polarizer



This is the only possible result. Its radius is the length of the bar from the previous graph. It's tilted in the direction perpendicular to the first. It's still polarized light. The ratio of this second intensity to the first intensity is not of interest. In light of the results from the first exercise the Area can just be divided.

$$\frac{A_{cascaded}}{A_{polarized}} = \frac{2\pi\left(r cos\theta\right)^2}{2\pi r^2} = cos^2\theta \tag{21}$$

This is Malus's law. Letting the first polarized light pattern radius be r and the second one be rcos(theta), since that is the parallel component, lead to this.

Light Detection

Light is detected or measured by atoms in a photomultiplier. Atoms are discrete objects with a discrete number of subatomic particles. As a consequence of their discrete nature, they will absorb or emit discrete amounts of energy as they change state. Atoms are noisy and unpredictable. That doesn't mean a classical model can't describe them but here it's more convenient to describe them in terms of probability. A photomultiplier works by having a material eject an electron upon irradiance. Classically ionization is a function of electric field strength. In light of Einstein's description of the photoelectric effect, in this case, it is known that the frequency of the source light is sufficient to ionize an electron in the detector, so the only remaining phenomena left to account for is the momentum change of the ejected electron, which is proportional to the electric field strength by Lenz law and Newton's second law. It is acknowledged that the probability of ionization has a factor of frequency, but in this case the source is monochromatic, so the factor would just vanish in the correlations regardless. This is an incomplete model, and hidden variables in the atom are assumed, but this expedites finding the solution to the problem at hand. Given all this, the probability of detection can be described by a Poisson process. In a Poisson process the signal to noise ratio is reciprocal of the coefficient of variation. The coefficient of variation for a Poisson process is the reciprocal of the square root of expected rate of occurrences

$$SNR = \sqrt{\lambda}, \lambda = E||, SNR = \sqrt{E||} \tag{22}$$

From equation 13 the field strength of polarized light is proportional to the incident light on the polarizer times the cosine of the angle between the polarizer and the incident light.

$$SNR = \sqrt{cos\theta} \tag{23}$$

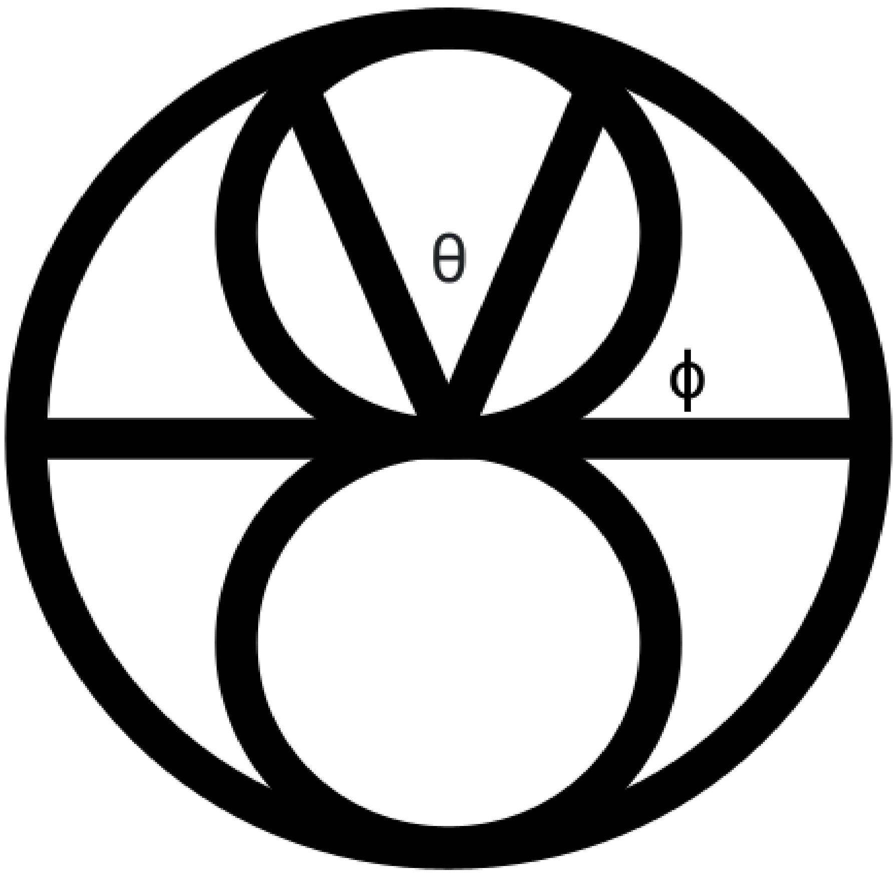
Bells Correlations

Bell is concerned with measurements of spin up or down, and how they are correlated. The correlation is as follows.

$$\frac{N_{++} - N_{+-} - N_{-+} + N_{--}}{N_{++} + N_{+-} + N_{-+} + N_{--}} = E(a, b) \tag{24}$$

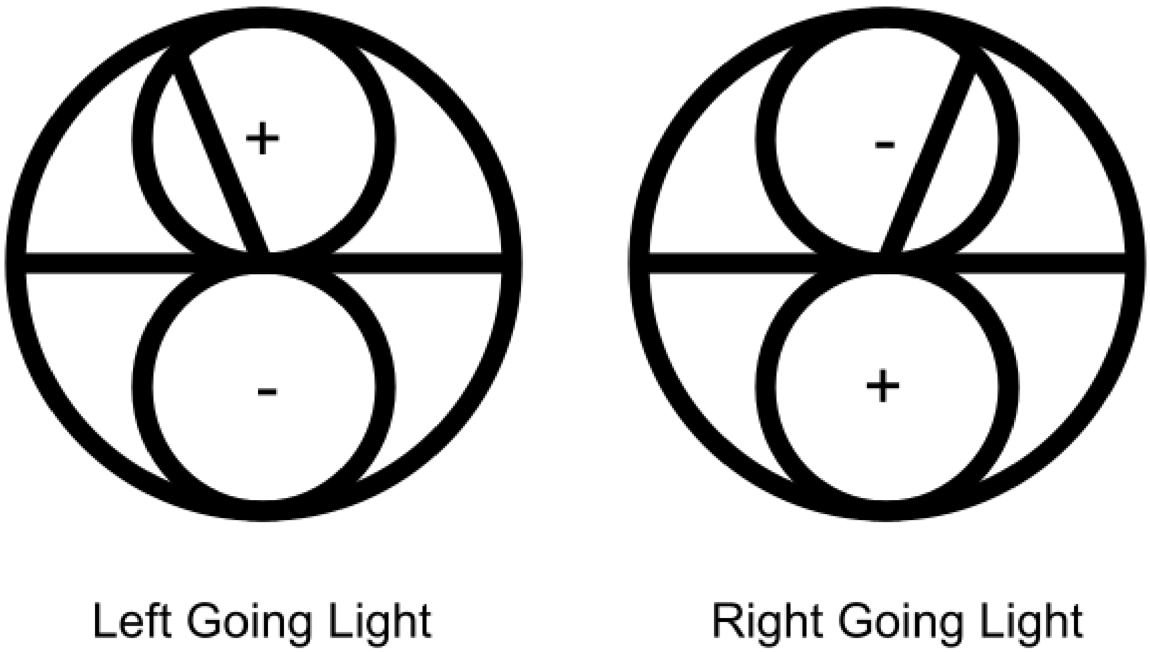
Here N denotes the number of trials that found photomultiplier detector hits corresponding to electric field parallel or anti-parallel to the polarizer. There are two polarizers, one has been set at angle a and the other at angle b. The positive or negative denotes whether the light was parallel or anti-parallel. The fact that there are two subscripts means that there are two separate trials happening in parallel, each one with a plane wave with a negative field than the other. Both detectors will receive polarized light, so illustrating the scenario with the polarizer angles a and b as bars is useful.

Fig. 10: Polarized Light Along Two Axis



In the previous figure theta is the angle between the two angles a and b. Phi is the angle the right bar both makes with the horizontal axis. It needs to be reiterated that the lobes here are anti-parallel. So in this state a detects positive spin and b detects negative spin.

Fig. 11: Opposite Lobes



This measurement, if both had a detector hit, would count for N+-. To find the total N over many trials an integral can be used, it represents all the different incident polarization angles. When phi is between zero and pi minus theta there will be a trial that gives another unit of N+-, if the results of that trial are not null. The probability of a successful trial is proportional to the field component in the parallel direction, as explained in the previous section. The sign of the sine will automatically add or subtract the parallel or anti-parallel term so the solution becomes the following.

$$N_{+-} = \int_{0}^{\pi-\theta} \sqrt{|sin(\phi)|}\sqrt{|sin(\phi + \theta)|}d\phi \qquad (26)$$

By inspection N-+ can be found in the region pi to two pi minus theta

$$N_{-+} = \int_{\pi}^{2\pi-\theta} \sqrt{|sin(\phi)|}\sqrt{|sin(\phi\ +\ \theta)|}d\phi \quad (27)$$

The remaining two regions will count for N++ and N--. From pi minus theta to pi both signs will be positive.

$$N_{++} = \int_{\pi-\theta}^{\pi} \sqrt{|sin(\phi)|}\sqrt{|sin(\phi\ +\ \theta)|}d\phi \quad (28)$$

$$N_{--} = \int_{2\pi-\theta}^{2\pi} \sqrt{|sin(\phi)|}\sqrt{|sin(\phi\ +\ \theta)|}d\phi \quad (29)$$

When evaluated this gives an very close approximation to the famous result. The correlation is approximately the negative of the angle between the polarizer angles.

$$E(\theta) \approx -\ cos\theta \quad (30)$$

This is the famous quantum result that leads to violation of Bells inequality. This is also the result of a rigorous classical derivation. What's left is to prove this result from simulation. It's claimed no classical machine can simulate the quantum result, because that would violate the CHSH inequality.

Simulating the Classical CHSH Violation

This is a causal, conservative, local, repeatable way to simulate the classical model given in this work for the results of Bell type experiments. There are no hidden variables. The only variables are the indecent polarization of the emitted light, and the angles of the polarizers. All processes are local and real. Simulation will be done with a computer. Computers are very good at simulating the cumulative distribution function of a uniformly distributed random variable. The probability density function of detection was given previously as the parallel component of the electric field.

$$\text{SNR}=P(D) = \sqrt{cos\theta} \tag{31}$$

A simple code written in octave will look like the following

```
function x=E(a,b)
 pp=0;
 nn=0;
 np=0;
 pn=0;
 N=10000;
 for k=1:N
   photon=2*pi*rand;
   A=sign(cos(photon-a))*(rand<sqrt(abs(cos(photon-a))));
   B=-sign(cos(photon-b))*(rand<sqrt(abs(cos(photon-b))));
   if(A==1 && B==1)
     pp=pp+1;
   endif
   if(A==-1 && B==-1)
     nn=nn+1;
```

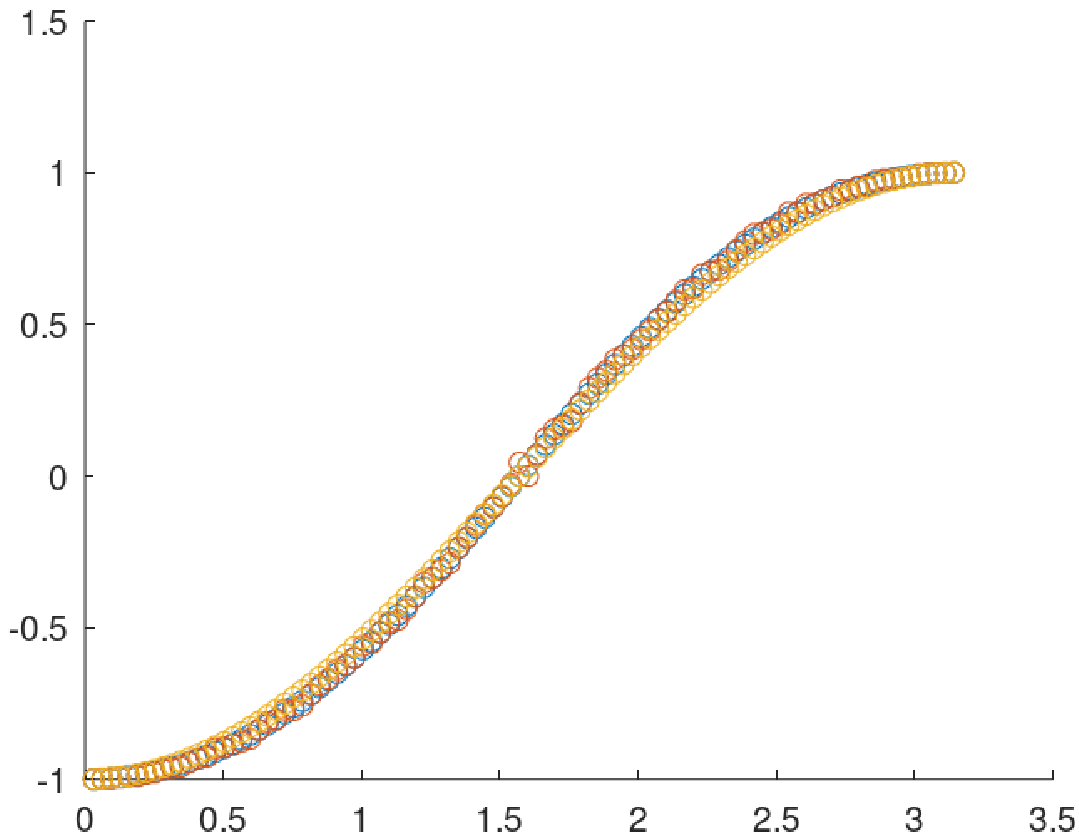
```
  endif
  if(A==-1 && B==1)
    np=np+1;
  endif
  if(A==1 && B==-1)
    pn=pn+1;
  endif
  endfor
 x=(pp+nn-pn-np)/(pp+nn+pn+np);
endfunction
```

In the previous code a function that calculates correlation is presented. There are a number of trials N. In each trial a linear polarization angle is chosen randomly. The polarizer angles a and b are passed to the function. The function computes the cosine of the angle between the polarizer and the incident polarized light. The sign is taken and used to add up correlations later, one sign is negated to simulate the zero net spin. The probability of being detected is calculated by seeing if a random variable is less than the electric field strength. If it doesn't pass then the sign variable is multiplied by zero. In this way the result of the trial can be -1,0, or 1. The sign represents a detection hit and if it was spin up or down. The zero represents no detection, so no adding into the correlation. There are the if blocks which increment the N counters for each case. In the program pp is N++, nn N--, np is N-+, and pn is N+-. The function returns the correlation.

Results

A graph of the correlation with respect to the relative angle between the polarizers follows. This matches the classical theory, the quantum theory and the experimental results.

Fig. 12: Simulation Results



Orange in the figure is the simulated correlation. Blue is the negative cosine of theta. Yellow is the calculation of the theoretical integral for the angle between detectors.

Conclusion

It's remarkable that an obviously classical process can recreate the correlations from the quantum model. It was claimed no physical process, not even in simulation, can recreate the correlation seen in the Bell type experiments. It was claimed that this level of correlation can only be the product of superluminal state communication between the two branches. That's not necessarily true. This casts serious doubt on the statistics of Bell.

Where Quantum Mechanics Went Wrong.

Sarcastically this could be a very long section. In terms of the Bell experiment theory, the problem is that, for the classical limit, omniscience of the detector is assumed. In a trial of a thousand one incident light angle will be one thousandth of two pi from being perpendicular to polarizer. Being a small angle, the resulting electromagnetic radiation will be one thousandth of the incident. How can anyone detect an electric field one thousandth of the value of one electron moving a few nanometers? There is, of course, a limit. When the derivation for the classical correlation limit is done, it's assumed that this isn't an issue, and any amount of electric field can be detected. That's not reasonable. When the fact of realistic detection is treated practically the classical limit is violated and the experimental results are found.

References

These are all more than 100 years old and don't seem to follow the modern publication indexing so I'll just give names.

1. Sir Issac Newton

2. Charles-Augustin de Coulomb

3. Thomas Young

4. James Clerk Maxwell

5. Étienne-Louis Malus

6. Siméon Denis Poisson

Ethics declarations

I declare there are no competing interests

Supplementary Information

Classical physics.