Synthetic Loss

Background

Having a myriad of high frequency, high quality, TDR data will lead to the following question. What to do with it? TDR yields a convincing profile of the impedance as a function of length along the trace. Designers and vendors of PCBs are not particularly interested in the impedance. It's a waypoint between the actual board and its performance. The performance of a board will be determined primarily by its loss as a function of frequency. It's assumed that deviations in impedance will lead to more loss. This dependence is not always clear. The relationship is not linear. Losing 1% of impedance control is not 1% more loss.

This paper describes a quick way to transform a TDR profile into a loss profile. This is not the insertion loss. It's a synthesized representation of the loss the channel would have if the signal propagates down the channel nominally and the only non-nominal loss is from the deviations in impedance particular to that trace on that PCB. Here the conductor loss and dielectric loss will have to be assumed to be nominal. In reality, there is a slight variance between traces and boards, in terms of conductor and dielectric loss, but that information is not available from a one terminal TDR measurement.

The Derivation

The TDR will give us a series of discrete impedance samples that represent the impedance down the length of the trace. The samples are evenly spaced. The system will look like the following.

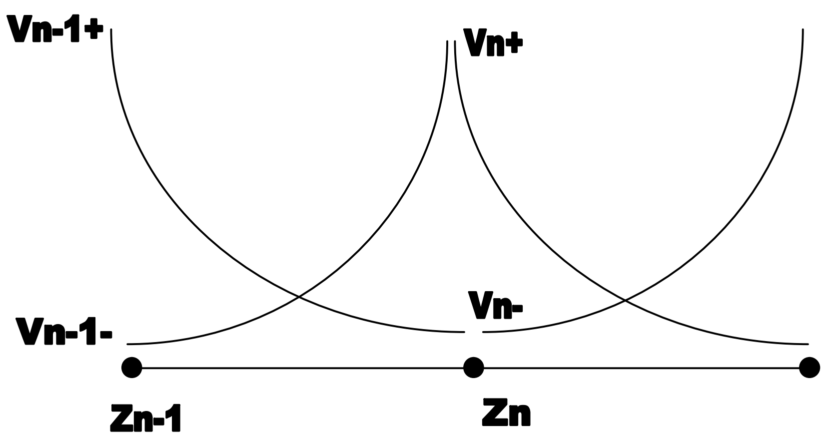


Figure 1. The system of impedance samples from the TDR

The system is composed of positive and negative traveling waves on a segment whose impedance is constant. All segments along the line will share a similar propagation constant. This leads to the following set of equations.

$$v_{n-1}^{+}e^{-\gamma d} \;\; + v_{n-1}^{-}e^{+\gamma dx} = v_n^{+} + v_n^{-}$$

$$\frac{v_{n-1}^{+}}{Z_{n-1}}e^{-\gamma dx} - \frac{v_{n-1}^{-}}{Z_{n-1}}e^{+\gamma dx} = \frac{v_n^{+}}{Z_n} - \frac{v_n^{-}}{Z_n}$$

The first equation is a form of Kirchhoff's law. The voltages superimpose at the boundary between the two regions. The previous wave propagated a length segment to be evaluated on the boundary. The second equation is Kirchhoff's current law. The currents superimpose. The difference is the reflected current is subtracted, since it has the opposite sense. After some algebraic manipulation, the system will take the following matrix form.

$$\begin{bmatrix} e^{-\gamma dx}\left(\frac{Z_{n-1}+Z_n}{2Z_{n-1}}\right) & e^{+\gamma dx}\left(\frac{Z_{n-1}-Z_n}{2Z_{n-1}}\right) \\ e^{-\gamma dx}\left(\frac{Z_{n-1}-Z_n}{2Z_{n-1}}\right) & e^{+\gamma dx}\left(\frac{Z_{n-1}+Z_n}{2Z_{n-1}}\right) \end{bmatrix} \begin{bmatrix} v_{n-1}^{+} \\ v_{n-1}^{-} \end{bmatrix} = \begin{bmatrix} v_n^{+} \\ v_n^{-} \end{bmatrix}$$

The advanced forward and backwards waves can be expressed in terms of the previous ones. This is important. In this way each segment (or each impedance sample) can be cascaded together to form a transfer function of the entire trace. As a consequence the entire trace can be characterized as follows.

$$\prod_{n=0}^{N} \begin{bmatrix} e^{-\gamma dx}\left(\frac{Z_{n-1}+Z_n}{2Z_{n-1}}\right) & e^{+\gamma dx}\left(\frac{Z_{n-1}-Z_n}{2Z_{n-1}}\right) \\ e^{-\gamma dx}\left(\frac{Z_{n-1}-Z_n}{2Z_{n-1}}\right) & e^{+\gamma dx}\left(\frac{Z_{n-1}+Z_n}{2Z_{n-1}}\right) \end{bmatrix} \begin{bmatrix} v_1^{+} \\ v_1^{-} \end{bmatrix} = \begin{bmatrix} v_2^{+} \\ v_2^{-} \end{bmatrix}$$

The product of all (N) the impedance transfer matrices will form a transfer function of a two port system representing the channel. The boundary conditions can also be applied. Assume the signal into the system is 1. Assume the signal traveling backwards at the last segment is 0. The product of all matrices will also be represented another way.

$$\prod_{n=0}^{N} \begin{bmatrix} e^{-\gamma dx}\left(\frac{Z_{n-1}+Z_n}{2Z_{n-1}}\right) & e^{+\gamma d}\left(\frac{Z_{n-1}-Z_n}{2Z_{n-1}}\right) \\ e^{-\gamma d}\left(\frac{Z_{n-1}-Z_n}{2Z_{n-1}}\right) & e^{+\gamma dx}\left(\frac{Z_{n-1}+Z_n}{2Z_{n-1}}\right) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} 1 \\ v_1^{-} \end{bmatrix} = \begin{bmatrix} v_2^{+} \\ 0 \end{bmatrix} \rightarrow \begin{matrix} a_{11} + a_{12}v_1^{-} = v_2^{+} \\ a_{21} + a_{22}v_1^{-} = 0 \end{matrix} \rightarrow \begin{matrix} v_2^{+} = \dfrac{a_{22}a_{11} - a_{12}a_{21}}{a_{22}} = s_{21} \\ v_1^{-} = \dfrac{-a_{21}}{a_{22}} = s_{11} \end{matrix}$$

The advanced reader will recognize the final algebra to convert the product of all matrices to the s-parameters is similar to the transformation from a T-parameter matrix to an s-parameter matrix. This is a consequence of the fact that the transfer matrix is the inverse of the T-parameter matrix, since the author defines the transfer matrix opposite in propagation as a traditional t-parameter matrix, for clarity.

Computational Complexity

The s-parameters for insertion and return loss can be recovered. This is a straight forward and quick approach too. Multiplying a few thousand 2X2 matrices is a trivial task for a computer. The computational complexity for more frequency samples or impedance samples will be linear. The time required for this calculation could be from tenths of seconds to seconds for reasonable resolution.

Sanity Checks

If the system is just two segments, the system breaks down to the following.

$$\begin{bmatrix} \left(\frac{Z_{n-1}+Z_n}{2Z_{n-1}}\right) & \left(\frac{Z_{n-1}-Z_n}{2Z_{n-1}}\right) \\ \left(\frac{Z_{n-1}-Z_n}{2Z_{n-1}}\right) & \left(\frac{Z_{n-1}+Z_n}{2Z_{n-1}}\right) \end{bmatrix} \begin{bmatrix} 1 \\ v_{n-1}^- \end{bmatrix} = \begin{bmatrix} v_n^+ \\ 0 \end{bmatrix}$$

After some algebra, the following can be derived

$$v_{n-1}^- = \frac{Z_n - Z_{n-1}}{Z_n + Z_{n-1}} = \Gamma, v_n^+ = \frac{2Z_n}{Z_n + Z_{n-1}} = 1 + \Gamma$$

The reflected wave takes the form of the well known reflection constant. The transmitted wave takes the form of the well known transmission coefficient.

If the system is just a long ideal trace with only nominal dielectric and conductor loss, the system will take the following form.

$$\begin{bmatrix} e^{-\gamma dx} & 0 \\ 0 & e^{+\gamma dx} \end{bmatrix}^N \begin{bmatrix} 1 \\ v_{n-1}^- \end{bmatrix} = \begin{bmatrix} v_n^+ \\ 0 \end{bmatrix}$$

Here N is the number of identical impedance samples. By a property of a diagonal matrix the following is found.

$$\begin{bmatrix} e^{-\gamma N} & 0 \\ 0 & e^{+\gamma N} \end{bmatrix} \begin{bmatrix} 1 \\ v_{n-1}^- \end{bmatrix} = \begin{bmatrix} v_n^+ \\ 0 \end{bmatrix}$$

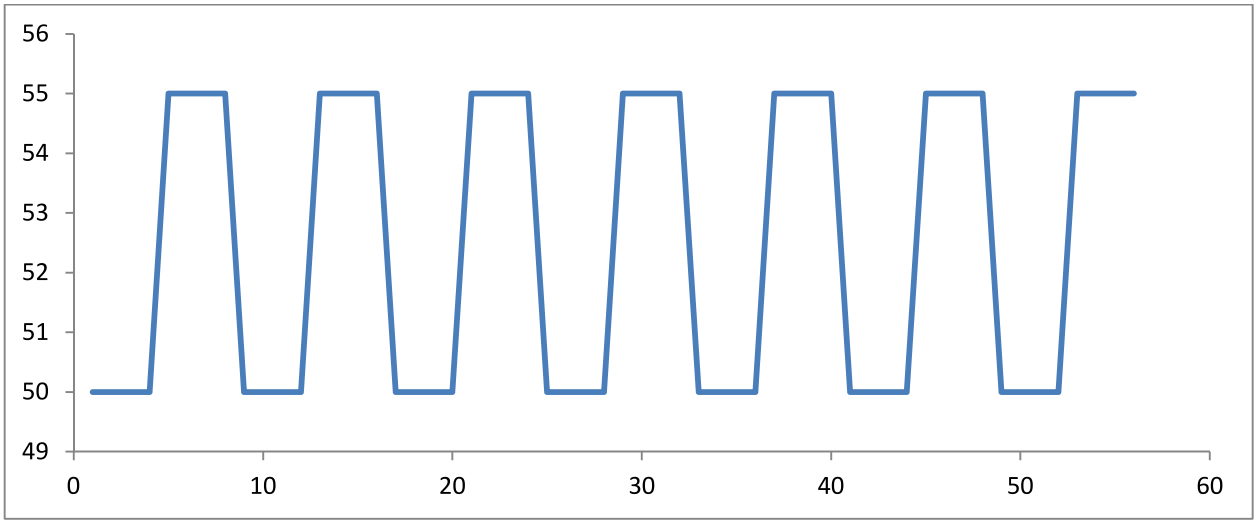
$$v_{n-1}^- e^{+\gamma N dx} = 0 \rightarrow \underline{v_{n-1}^- = 0}, \underline{v_n^+ = e^{-\gamma l}}$$

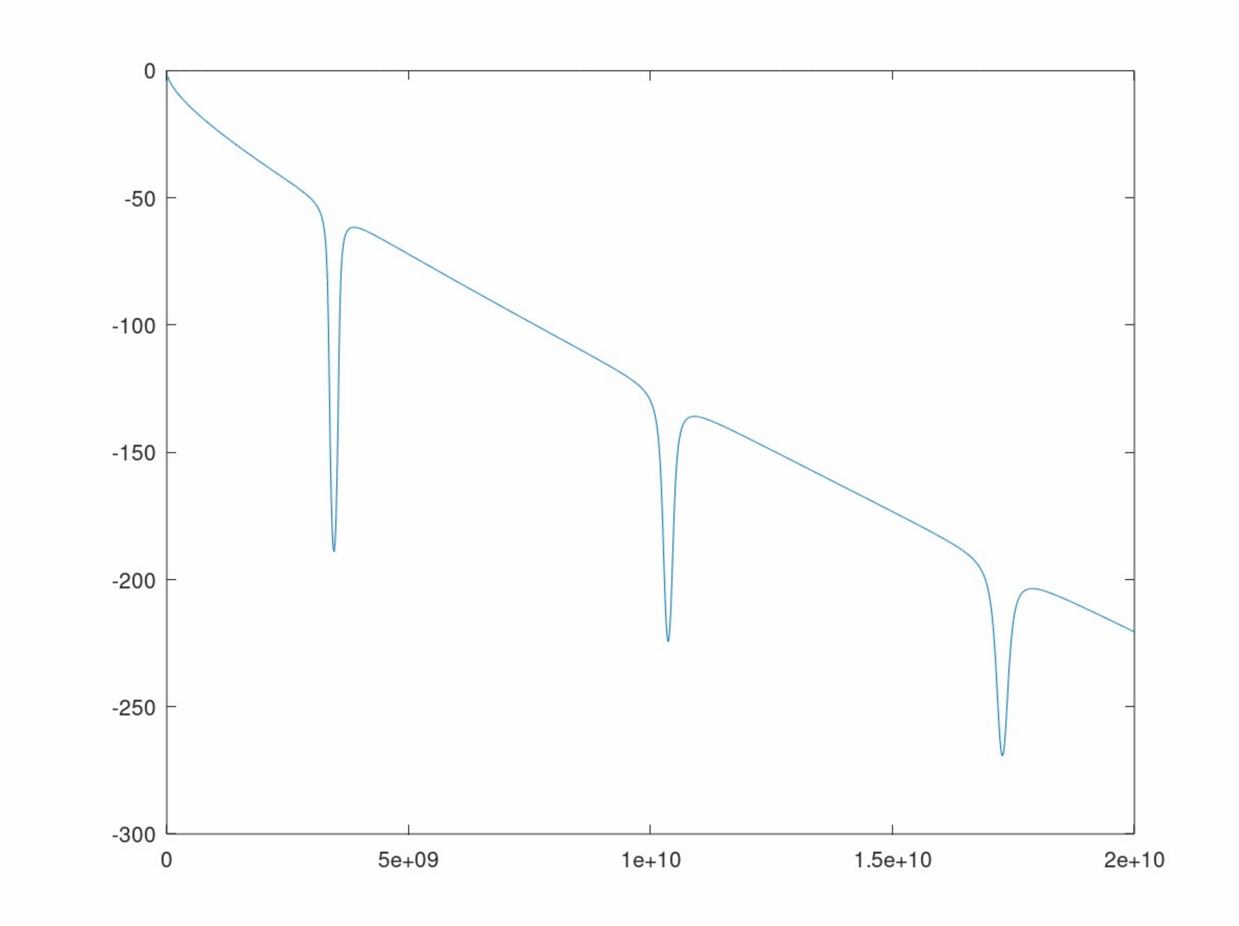
There is no reflection and what is transmitted is the nominal signal attenuated by conductor and dielectric loss.

A system can also be created for a periodic medium rather easily. The system will go from 50 to 55 ohms periodically hundreds of times. The system will be composed of the following three matrices.

$$\begin{bmatrix} e^{-\gamma dx}\left(\frac{105}{100}\right) & -e^{+\gamma dx}\left(\frac{5}{100}\right) \\ -e^{-\gamma dx}\left(\frac{5}{100}\right) & e^{+\gamma dx}\left(\frac{105}{100}\right) \end{bmatrix}, \begin{bmatrix} e^{-\gamma dx}\left(\frac{105}{110}\right) & e^{+\gamma dx}\left(\frac{5}{110}\right) \\ e^{-\gamma dx}\left(\frac{5}{110}\right) & e^{+\gamma d}\left(\frac{105}{110}\right) \end{bmatrix}, \begin{bmatrix} e^{-\gamma d} & 0 \\ 0 & e^{+\gamma dx} \end{bmatrix}$$

Respectively, these are the matrix for the impedance going up, the impedance going down, and remaining constant. Multiplying these together in the following order will lead to the insertion loss profile that follows.

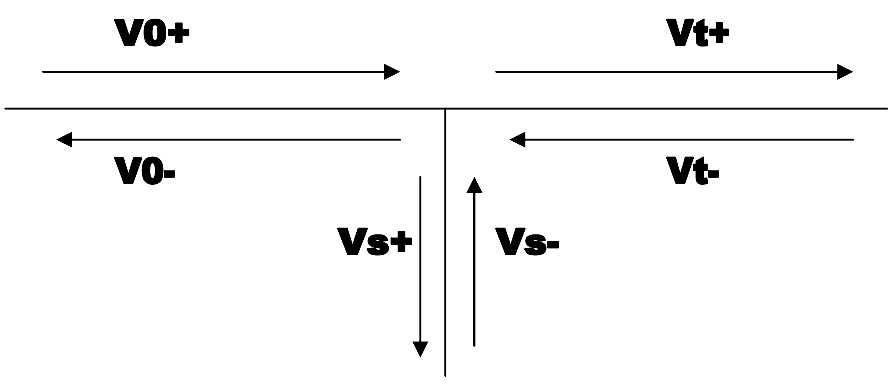




This is typical Bloch type propagation. This proves that while the calculation is simple, the results will take into account all the complexity of timing and resonance. This calculation took 1 second in Octave.

Stubs

Stubs are an important part of a PCB manufacturing process. A stub can cause loss orders of magnitude more severe than the most egregious impedance deviations. This method can be extended to include the effect of stubs. The effect of the stub can only be included if the length of the stub is known. With a quality TDR and a good high-speed connection a stub length is proportional to the length of the half impedance section in the launch area. A breakdown of the derivation of the stub section follows. There is a channel and a branch that the signal can propagate down.



The s section is the stub branch. The t section is the transmission line channel. The 0 section is the launch. The system can be described mathematically in the following manner, using kirchoffs voltage and current laws.

$$v_0^+ + v_0^- = v_t^+ + v_t^-$$

$$v_0^+ + v_0^- = v_s^+ + v_s^-$$

$$v_0^+ - v_0^- = v_t^+ - v_t^- + v_s^+ - v_s^-$$

$$v_s^+ e^{-\gamma} - v_s^- e^{+\gamma s} = 0$$

After some algebra the system can form a matrix equation. S is the stub length

$$\begin{bmatrix} \left(1 - \frac{1}{2}\tanh(\gamma s)\right) & -\frac{1}{2}\tanh(\gamma s) \\ \frac{1}{2}\tanh(\gamma s) & \left(1 + \frac{1}{2}\tanh(\gamma s)\right) \end{bmatrix} \begin{bmatrix} v_0^+ \\ v_0^- \end{bmatrix} = \begin{bmatrix} v_t^+ \\ v_t^- \end{bmatrix}$$

As before, by inspection, the advanced transmission and reflection can be expressed in terms of the previous. The system is a t-parameter segment that can be cascaded with the previously derived ones. As a sanity check the system is inspected with a zero length stub.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_0^+ \\ v_0^- \end{bmatrix} = \begin{bmatrix} v_t^+ \\ v_t^- \end{bmatrix}$$

The launch portion of the channel becomes the identity matrix, so it vanishes. The tangent function can reach undefined values. The function should be examined there.

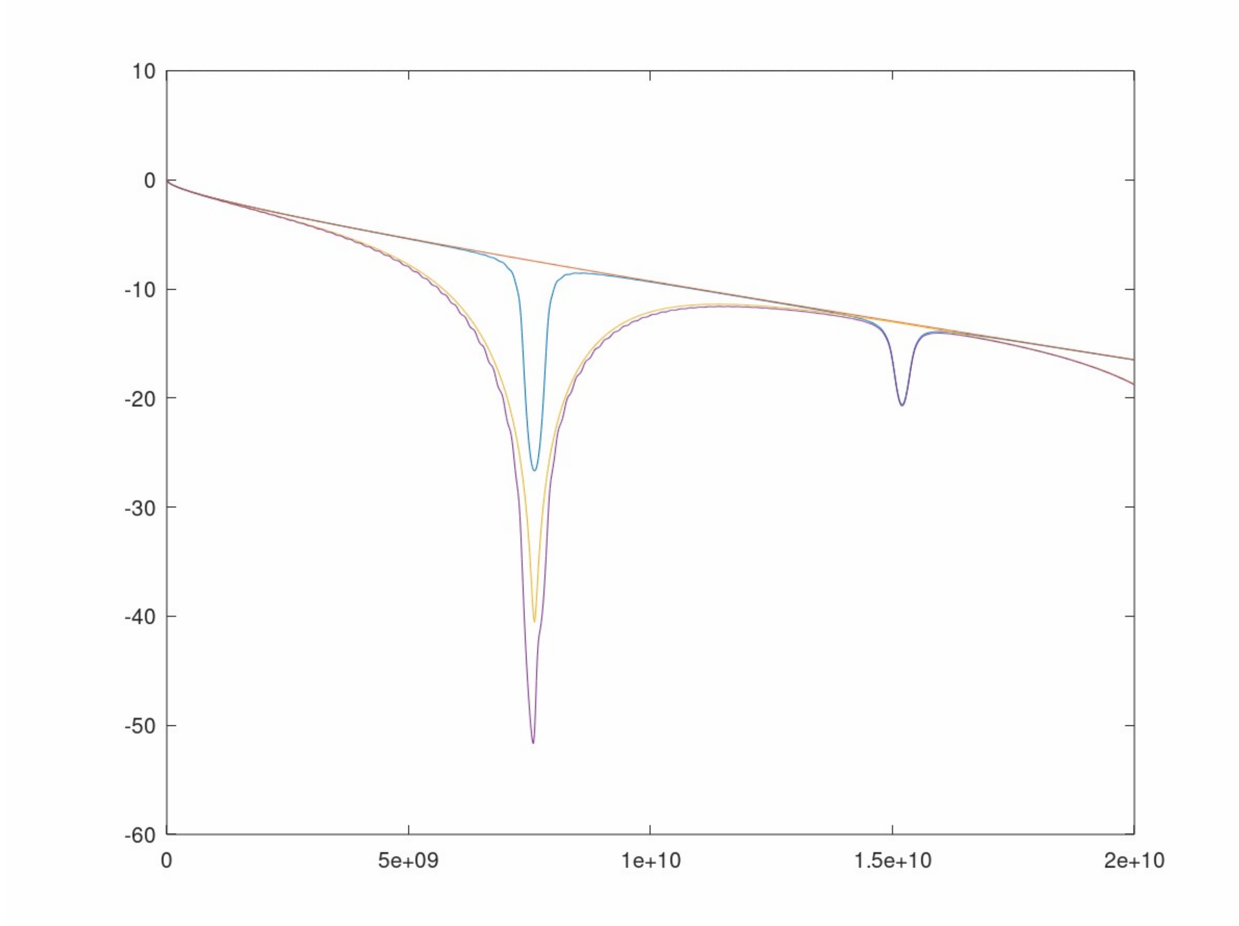
$$\gamma s = i\frac{\pi}{2}$$

$$\begin{bmatrix} -\infty & -\infty \\ \infty & \infty \end{bmatrix} \begin{bmatrix} v_0^+ \\ v_0^- \end{bmatrix} = \begin{bmatrix} v_t^+ \\ v_t^- \end{bmatrix}, \underline{v_0^- = -1}, \underline{v_t^+ = 0}, \underline{s = \frac{\lambda}{4}}$$

This is the classic stub result. A stub will block at a frequency that corresponds to a quarter wavelength of the stub. Passing these sanity checks, a system with a variable impedance and stub length can be expressed as follows.

$$\prod_{n=0}^{N} \begin{bmatrix} e^{-\gamma dx}\left(\frac{Z_{n-1} + Z_n}{2Z_{n-1}}\right) & e^{+\gamma dx}\left(\frac{Z_{n-1} - Z_n}{2Z_{n-1}}\right) \\ e^{-\gamma dx}\left(\frac{Z_{n-1} - Z_n}{2Z_{n-1}}\right) & e^{+\gamma dx}\left(\frac{Z_{n-1} + Z_n}{2Z_{n-1}}\right) \end{bmatrix} \begin{bmatrix} 1 - \frac{1}{2}\tanh(\gamma s) & -\frac{1}{2}\tanh(\gamma s) \\ \frac{1}{2}\tanh(\gamma s) & 1 + \frac{1}{2}\tanh(\gamma s) \end{bmatrix}$$

This is the complete transfer function for a channel with a stub in the launch. The complexity of the expression has not grown significantly. A plot of this expression evaluated follows.



The red line is no impedance deviation and no stub. The blue is a periodic medium only. The yellow is a stub only. The purple is a stub and a periodic medium.

Further Work

This was part of my master plan to treat TDR data. I've outlined a quick, simple, and easy to implement way to transform TDR data into loss in the frequency domain. What remains to be done is less complex and more straight forward. To transform loss in the frequency domain to an eye diagram in the time domain is a simple multiplication by the Fourier transform of a step function or pseudo-random bit sequence. Then inverse Fourier transform it. Then a mask can be applied to check for channel pass/fail. This paper represents most of the heavy lifting towards being able to treat TDR data in a scientific, mathematical way that yields a definitive pass/fail for each channel.