Simple Functions for Impedance Tolerances

Background

The customer likes to specify a differential impedance tolerance for differential channels, but what if we can only measure the single-ended impedance on one leg of the differential. What relationship exists between the differential and single-ended impedance.

Derivation

Start with the expressions for the even and odd impedance

$$Z_e = \sqrt{\frac{L_{11} + L_{12}}{C_{11} + C_{12}}}, Z_d = \sqrt{\frac{L_{11} - L_{12}}{C_{11} - C_{12}}}$$

Notice the odd impedance is just a ratio of a subtraction of inductance and capacitances

The expression for the error of the inductance and capacitance follows for the odd impedance

$$\delta L = \sqrt{(\delta L_{11})^2 + (\delta L_{12})^2}, \delta C = \sqrt{(\delta C_{11})^2 + (\delta C_{12})^2}$$

The expression for the error propagation in the even capacitance and inductance follows

$$\delta L = \sqrt{(\delta L_{11})^2 + (\delta L_{12})^2}, \delta C = \sqrt{(\delta C_{11})^2 + (\delta C_{12})^2}$$

Notice they are the same. If the customer specifies an error tolerance in the differential impedance, which is twice the odd impedance, then they are also specifying an uncertainty in the even impedance, since the even and odd impedance are functions of the same physical parameters just added instead of subtracted, but the error propagates the same. So the following is true, neglecting correlation.

$$\delta Z_e = \delta Z_d$$

Now lets look at the expression for single-ended impedance

$$Z_{SE} = \frac{Z_e + Z_d}{2}$$

Since the single-ended impedance is just an addition of the even and odd, the following is true for the error. Doing both cases for uncorrelated(corr=0, no correlation) and correlated(corr=1, perfect correlation) yields the following

$$\delta Z_{SE,corr=0} = \sqrt{\left(\delta \frac{1}{2} Z_e\right)^2 + \left(\delta \frac{1}{2} Z_d\right)^2}$$

$$\delta Z_{SE,corr=} \quad = \sqrt{\left(\delta \frac{1}{2} Z_e\right)^2 + \left(\delta \frac{1}{2} Z_d\right)^2 + 2\delta \frac{1}{2} Z_e \delta \frac{1}{2} Z_d}$$

$$\delta Z_{SE,corr=0} = \sqrt{\frac{1}{4}(\delta Z_d)^2 + \frac{1}{4}(\delta Z_d)^2} \rightarrow \sqrt{\frac{1}{2}(\delta Z_d)^2}$$

$$\delta Z_{SE,corr=1} = \sqrt{\frac{1}{2}(\delta Z_d)^2 + \frac{1}{2}(\delta Z_d)^2} \rightarrow \delta Z_d$$

$$\delta Z_{SE,corr=} \quad = \frac{\sqrt{2}}{2} \delta Z_d$$

$$\delta Z_{SE,corr=} \quad = \delta Z_d$$

When the even and odd impedance are not correlated, the error in the single-ended impedance is 70% of the error in the odd impedance. This is a consequence of the single-ended impedance being an average in the even and odd. When numbers are averaged the error is reduced. When there is perfect correlation the single-ended error is the same as the odd. Finishing, the odd impedance is converted back to differential.

$$\delta Z_{SE,corr=1} = \frac{\sqrt{2}}{2} \delta Z_d, \delta Z_{SE,corr=} \quad = \delta Z_d, Z_{diff} = 2Z_d, \frac{1}{2} \delta Z_{diff} = \delta Z_d$$

$$\underline{\delta Z_{SE,corr=} \quad = \frac{\sqrt{2}}{4} \delta Z_{diff}}, \underline{\delta Z_{SE,corr=1} = \frac{1}{2} \delta Z_{diff}}$$

Conclusion

The error in the single-ended impedance is 35% the differential error, in general, since they are derived from the same physical parameters, but only differ by sign. This result neglects correlation in any impedances. From the perspective of having a perfect correlation, the single-ended impedance would be 50% of the differential impedance. In any case of the correlation being between 0 and 1 the single-ended impedance will be less than half of the differential. This is an upper bound. Since we will only experience more loss reducing the error tolerance, and we can’t argue for a more lenient tolerance, assigning the customers error in percent to the single-ended impedance is the only rational solution.